① $(a+b)^5 = 1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$

So for $(8+\sqrt{2})^5$, we substitute $a=8$ and $b=\sqrt{2}$:

$$(8+\sqrt{2})^5 = 8^5 + 5(8)^4(\sqrt{2}) + 10(8)^3(\sqrt{2})^2 + 10(8)^2(\sqrt{2})^3 + 5(8)(\sqrt{2})^4 + (\sqrt{2})^5$$

$$= 32768 + 20480\sqrt{2} + 10240 + 1280\sqrt{2} + 160 + 4\sqrt{2}$$

$$= 43168 + 21764\sqrt{2}$$

---

② $(a+b)^7 = a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7$

$$\therefore (1+\sqrt{3})^7 = 1 + 7\sqrt{3} + 63 + 105\sqrt{3} + 315 + 189\sqrt{3} + 189 + 27\sqrt{3}$$

$$= 568 + 328\sqrt{3}$$

---

③ $(x+6)^4 = x^4 + (4)(x^3)(6) + 6(x^2)(6^2) + 4(x)(6^3) + 6^4$

$$= x^4 + 24x^3 + 216x^2 + 864x + 1296$$

---

④ $\left(x+\frac{3}{4}\right)^7 = x^7 + 7x^6\left(\frac{3}{4}\right) + 21x^5\left(\frac{3}{4}\right)^2 + 35x^4\left(\frac{3}{4}\right)^3 + 35x^3\left(\frac{3}{4}\right)^4 + 21x^2\left(\frac{3}{4}\right)^5 + 7x\left(\frac{3}{4}\right)^6 + \left(\frac{3}{4}\right)^7$

$$= x^7 + \frac{21x^6}{4} + \frac{169x^5}{16} + \frac{945x^4}{64} + \frac{2835x^3}{256} + \frac{5103x^2}{1024} + \frac{5103x}{4096} + \frac{2187}{16384}$$

5. $(3x+\sqrt{5})^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$$= (3x)^3 + 3(3x)^2(\sqrt{5}) + 3(3x)^1(\sqrt{5})^2 + (\sqrt{5})^3$$

$$= 27x^3 + 27\sqrt{5}\,x^2 + \cancel{45}\;45x + 5\sqrt{5}$$

6. $(x+3\sqrt{5})^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$$= x^4 + 4(x)^3(3\sqrt{5}) + 6(x)^2(3\sqrt{5})^2 + 4x(3\sqrt{5})^3 + (3\sqrt{5})^4$$

$$= x^4 + 12\sqrt{5}\,x^3 + 270x^2 + 540\sqrt{5}\,x + 2025$$

7) 8^{th} term in $(3+\sqrt{5})^{11}$

n	1	2	3	4	5	6	7	8
Pa	1	11	55	165	330	462	330	165
a	11	10	9	8	7	6	5	4
b	0	1	2	3	4	5	6	7

$$t_8 = 165(3)^4(\sqrt{5})^7$$
$$= 165(81)(125\sqrt{5})$$
$$= 1{,}670{,}625\sqrt{5}$$

---

8) 12^{th} term of $(3x+\sqrt{3})^{17}$

n	1	2	3	4	5	6	7	8	9	10	11	12
Pa	1	17	136	680	2380	6188	12376	19448	24310	24310	19448	12376
a	17	16	15	14	13	12	11	10	9	8	7	6
b	0	1	2	3	4	5	6	7	8	9	10	11

$$t_{12} = 12376(3x)^6(\sqrt{3})^{11}$$
$$= 12376(729x^6)(243\sqrt{3})$$
$$= 2{,}192{,}371{,}272\sqrt{3}\,x^6$$

---

(9) 6^{th} term in $\left(4x^2 + \frac{2}{3}\right)^{10}$

n	1	2	3	4	5	6
Pa	1	10	45	120	210	252
a	10	9	8	7	6	5
b	0	1	2	3	4	5

$$t_6 = 252\left(4x^2\right)^5\left(\frac{2}{3}\right)^5$$

$$= 252\left(1024x^{10}\right)\left(\frac{32}{243}\right)$$

$$= \frac{8257536x^{10}}{243}$$

---

(10) 7^{th} term in $\left(2x^2 + \sqrt{3}\right)^{11}$

n	1	2	3	4	5	6	7
Pa	1	11	55	165	330	462	462
a	11	10	9	8	7	6	5
b	0	1	2	3	4	5	6

$$t_7 = 462\left(2x^2\right)^5\left(\sqrt{3}\right)^6$$

$$= 462\left(32x^{10}\right)\left(27\right)$$

$$= 399{,}168x^{10}$$

(11) 9th term of $\left(3x^2 + \frac{1}{\sqrt{3}}\right)^{10}$

n	1	2	3	4	5	6	7	8	9
Pa	1	10	45	120	210	252	210	120	45
a	10	9	8	7	6	5	4	3	2
b	0	1	2	3	4	5	6	7	8

$$9\text{th term} = 45\left(3x^2\right)^2\left(\frac{1}{\sqrt{3}}\right)^8$$

$$= 45\left(9x^4\right)\left(\frac{1}{81}\right)$$

$$= 5x^4$$

---

(12) 11th term of $\left(\frac{2}{\sqrt{5}} - 3x\right)^{12}$

n	1	2	3	4	5	6	7	8	9	10	11
Pa	1	12	66	220	495	792	924	792	495	220	66
a	12	11	10	9	8	7	6	5	4	3	2
b	0	1	2	3	4	5	6	7	8	9	10

$$t_{11} = 66\left(\frac{2}{\sqrt{5}}\right)^2\left(-3x\right)^{10}$$

$$= 66\left(\frac{4}{5}\right)\left(59049x^{10}\right)$$

$$= \frac{15588936x^{10}}{5}$$

(13) 5th term in $\left(7x - \frac{4}{1+2\sqrt{2}}\right)^7$

n	1	2	3	4	5
Pa	1	7	21	35	35
a	7	6	5	4	3
b	0	1	2	3	4

$$\text{Fifth term} = 35\,(7x)^3\left(-\frac{4}{1+2\sqrt{2}}\right)^4$$

$$= 35\,(343x^3)\left(-\frac{256}{113+72\sqrt{2}}\right)$$

$$= \frac{-3073280x^3}{113+72\sqrt{2}}$$

$\frac{1}{b^2}$

	1	$+2\sqrt{2}$
1	1	$+2\sqrt{2}$
$+2\sqrt{2}$	$+2\sqrt{2}$	$+8$

$\frac{1}{b^3}$

	9	$+4\sqrt{2}$
1	9	$+4\sqrt{2}$
$+2\sqrt{2}$	$+18\sqrt{2}$	$+16$

$\frac{1}{b^3}$

	25	$+22\sqrt{2}$
1	25	$+22\sqrt{2}$
$+2\sqrt{2}$	$+50\sqrt{2}$	$+88$

(14) 4th term in $\left(\frac{3}{5} - \frac{\sqrt{3}}{2}\right)^{12}$

n	1	2	3	4
Pa	1	12	66	220
a	12	11	10	9
b	0	1	2	3

$$t_4 = 220\left(\frac{3}{5}\right)^{9}\left(-\frac{\sqrt{3}}{2}\right)^{3}$$

$$= 220\left(\frac{19683}{1953125}\right)\left(-\frac{3\sqrt{3}}{8}\right)$$

$$= \frac{-12990780\sqrt{3}}{15625000}$$

$$= -\frac{649539\sqrt{3}}{781250}$$

(15) 7th term in $\left(\frac{5x}{2} - \frac{3}{\sqrt{7}}\right)^8$

n	1	2	3	4	5	6	7
Pa	1	8	28	56	70	56	28
a	8	7	6	5	4	3	2
b	0	1	2	3	4	5	6

$28a^2b^6$

$$t_7 = 28\left(\frac{5x}{2}\right)^2\left(\frac{-3}{\sqrt{7}}\right)^6$$

$$= 28\left(\frac{25x^2}{4}\right)\left(\frac{729}{343}\right)$$

$$= \frac{510300x^2}{1372}$$

$$= \frac{127575x^2}{343}$$

---

(16) $n = 3 + 2 = 5$

x	5	4	3	2	1	0
Pa	1	5	10	10	5	1

$$\frac{80x^3y^2}{10} = 8x^3y^2$$

$$\sqrt[3]{8x^3} = 2x$$

$$\therefore (mx+y)^n = (2x+y)^5$$

$x^4y = t_2$

$t_2 = 5(2x)^4(y)$

$= 80x^4y$

So the co-efficient is 80.

(17) $n = 3 + 4 = 7$

x	7	6	5	4	3	2	1	0
Pa	1	7	21	35	35	21	7	1

$$\frac{2835x^4y^3}{35} = 81x^4y^3$$

$$\sqrt[4]{81x^4} = 3x$$

$$\therefore (mx+y)^n = (3x+y)^7$$

$$t = 7(3x)^6y = 5103x^6y$$

So the coefficient is 5103.

---

(18) $n = 3 + 1 = 4$

x	4	3	2	1	0
Pa	1	4	6	4	1

$$\frac{-64x^3y}{4} = -16x^3y$$

$$\sqrt[3]{-16x^3} = -2.5198421x$$

$$\therefore (mx+y)^n = (-2.5198421x+y)^4$$

x^2y^2

$$6(-2.5198421x)^2(y^2) = -38.09762525x^2y^2$$

So the co-efficient is -38.09762525

(19) $n = 4 + 4 = 8$

x	8	7	6	5	4	3	2	1	0
Pa	1	8	28	56	70	56	28	8	1

$$\frac{43750x^4}{70} = 625x^4$$

$$\sqrt[4]{625x^4} = 5x$$

$$\therefore (mx+y)^n = (5x+y)^8$$

x^6y^2 term : $28(5x)^6(y)^2 = 437{,}500x^6y^2$

So 437,500 is the co-efficient.

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(20) $n = 3 + 7 = 10$

x	10	9	8	7	6	5	4	3	2	1	0
Pa	1	10	45	120	210	252	210	120	45	10	1

$$\frac{61440x^3}{120} = 512x^3$$

$$\sqrt[3]{512x^3} = 8x$$

$$\therefore (mx+y)^n = (8x+y)^{10}$$

x^ky^{k-2} So we have $k + k - 2 = 10$

$k + k = 12$

$k = 6$

x^6y^4

$$210(8x)^6y^4 = 55050240x^6y^4$$

So the co-efficient is 55,050,240